Simultaneous equations

Questions usually say 'solve the simultaneous equations' or solve simultaneously, but they occasionally ask 'find where these two lines intersect' All mean that we are looking for a pair of coordinates which work in both equations.

Example: $2x + 3y = 7$ **1**
$3x + 5y = 18$ **2**

Label the equations 1 and 2

1 x 3 → $6x + 9y = 21$ **3**
2 x 2 → $6x + 10y = 36$ **4**

Multiply the equations so that we have the same amount of x or y. This is worth 1 mark.

Same sign
Take away
Opposite sign
Plus

Because we have 6x in both equations and they are both positive, we need to subtract one equation from the other.

3 – 4 $6x + 9y = 21$ **3**
$6x + 10y = 36$ **4**
$-y = -15$ so $y = 15$

Next we substitute this value into one of the original equations

$2x + 3y = 7$
$2x + 45 = 7$
$2x = -38$ so $x = -19$

Example: $7x – 2y = -20$ **1**
$3x + 4y = 6$ **2**

Label the equations 1 and 2

1 x 2 → $14x - 4y = -40$ **3**

Multiply the equations so that we have the same amount of x or y.

Same sign
Take away
Opposite sign
Plus

We now have 4y in both equations. One is positive and one negative, so we need to add the equations.

2 + **3** $3x + 4y = 6$ **2**
$14x - 4y = -40$ **3**
$17x = -34$ so $x = -2$

Next we substitute this value into one of the original equations

$3x + 4y = 6$
$-6 + 4y = 6$
$4y = 12$ so $y = 3$

Questions

Q1. Solve the simultaneous equations

$$4x + y = 25$$
$$x - 3y = 16$$

(Total for Question is 3 marks)

Q2. Solve the simultaneous equations

$$3x - 4y = 8$$
$$9x + 5y = -1.5$$

(Total for Question is 3 marks)

Q3. Solve the simultaneous equations

$$3x + 4y = 5$$
$$2x - 3y = 9$$

(Total for Question is 4 marks)

Q4. Solve the simultaneous equations

$$4x + 7y = 1$$
$$3x + 10y = 15$$

(Total for Question is 4 marks)

Q5. Solve the simultaneous equations

$$5x + 2y = 11$$
$$4x - 3y = 18$$

(Total for Question is 4 marks)

Q6. Solve the simultaneous equations

$$3x + 2y = 4$$
$$4x + 5y = 17$$

(Total for Question is 4 marks)

Q7.
* A and B are straight lines.
Line A has equation $2y = 3x + 8$
Line B goes through the points (−1, 2) and (2, 8)

Do lines A and B intersect?
You must show all your working.

(Total for Question is 3 marks)

Harder simultaneous equations

More difficult simultaneous equations involve quadratics and need to be solved using **substitution**. This type of question gives 4 answers, rather than 2 in normal simultaneous equations.

Example: Solve the simultaneous equations

$$x^2 + y^2 = 100$$
$$x - y = 2$$

Step 1- rearrange the linear equation if necessary. X or y needs to be the subject

$$x = y + 2$$

Step 2- substitute into the quadratic and simplify

$$(y + 2)^2 + y^2 = 100$$
$$y^2 + 4y + 4 + y^2 = 100$$
$$2y^2 + 4y - 96 = 0$$
$$y^2 + 2y - 48 = 0$$

Step 3- Solve by factorising or using the formula

$(y + 8)(y – 6) = 0$

y = -8 or y = 6

Step 4- Substitute your two values into the linear equation to find the other solutions

$x = y + 2$

When y = -8, $x = -8 + 2 =$ **-6**

When y = 6, $x = 6 + 2 =$ **8**

Another example: Solve the simultaneous equations

$y = x^2 – 3x + 4$

$y – x = 1$

Step 1- rearrange the linear equation if necessary. X or y needs to be the subject

$y = 1 + x$

Step 2- substitute into the quadratic and simplify

$1 + x = x^2 – 3x + 4$

$x^2 – 4x + 3 = 0$

Step 3- Solve by factorising or using the formula

$(x – 1)(x – 3) = 0$

x = 1 or x = 3

Step 4- Substitute your two values into the linear equation to find the other solutions

$y = 1 + x$

When x = 1, $y = 1 + 1 =$ **2**

When x = 3, $y = 1 + 3 =$ **4**

Questions

1. $y = x^2 + 7x - 2$
   $y = 2x - 8$

2. $x^2 + y^2 = 8$
   $y = x + 4$

3. $y = x^2$
   $y = x + 2$

4. $x^2 + y^2 = 5$
   $x - 2y = 5$

5. Solve the equations

$$x^2 + y^2 = 36$$
$$x = 2y + 6$$

(Total for Question is 5 marks)

6. Solve the simultaneous equations

$$x^2 + y^2 = 25$$
$$y = 2x + 5$$

x = ... and y = ...

or

x = ... and y = ...

(Total for Question is 6 marks)

7. Solve the simultaneous equations

$$x^2 + y^2 = 9$$
$$x + y = 2$$

Give your answers correct to 2 decimal places.

x = ... y = ...

or x = ... y = ...

(Total for Question is 6 marks)

Answers

Q1.

Answer	Mark	Notes
$x = 7$ $y = -3$	3	M1 for correct process to eliminate one variable (condone one arithmetic error) M1 (dep) for substituting found value in one of the equations or appropriate method after starting again (condone one arithmetic error) A1 for $x = 7$ and $y = -3$

Q2.

$x = \frac{2}{3}$ $y = -1\frac{1}{2}$	3	M1 for correct process to eliminate either x or y (condone one arithmetic error) M1 (dep on 1st M1) for correct substitution of their found variable or other acceptable method A1 cao for both $x = \frac{2}{3}$ and $y = -1\frac{1}{2}$ oe SC: B1 for $x = \frac{2}{3}$ or $y = -1\frac{1}{2}$ oe NB: for $\frac{2}{3}$ accept working to 2 dp: 0.67 or 0.66 or better

Q3.

Working	Answer	Mark	Notes
$6x + 8y = 10$ $6x - 9y = 27$ $y = -1$ $3x - 4 = 5$ $3x = 9$ $x = 3$	$x = 3, y = -1$	4	M1 for a correct process to eliminate either variable (condone one arithmetic error) A1 cao for either x or y M1 (dep on M1) for correct substitution of found value into one of the equations or appropriate method after starting again (condone one arithmetic error) A1 cao

Q4.

Working	Answer	Mark	Notes
$12x + 21y = 3$ $12x + 40y = 60$ $19y = 57$ $y = 3$ $3x + 10 \times 3 = 15$ $3x = -15$	$x = -5, y = 3$	4	M1 for a correct process to eliminate either x or y or rearrangement of one equation leading to substitution (condone one arithmetic error) A1 for either $x = -5$ or $y = 3$ M1 (dep) for correct substitution of their found value

Q5.

Answer	Mark	Notes
$x = 3$ $y = -2$	4	M1 for coefficients of x or y the same followed by correct operation (condone one arithmetic error) A1 cao for first solution M1 (dep on M1) for correct substitution of found value into one of the equations or appropriate method after starting again (condone one arithmetic error) A1 cao for second solution

Q6.

Answer	Mark	Notes
$x = -2$ $y = 5$	4	M1 for a correct process to eliminate either x or y or leading to substitution (condone one arithmetic error) A1 for either $x = -2$ or $y = 5$ M1 (dep) for correct substitution of their found value A1 cao SC If M0 scored B1 for $y = -2$ and $x = 5$

Q7.

Answer	Mark	Notes
Yes with explanation	3	M1 For **Line A**: writes equation as $y = 1.5x + 4$ or gives the gradient as 1.5 or constant term of 4 OR for **Line B**: shows a method which could lead to finding the gradient or gives the gradient as 2 or constant term of 4 or calculates a sequence of points including (0,4) or writes equation of line as $y = 2x + 4$ M1 Shows correct aspects relating to an aspect of Line A and an aspect of Line B that enables some comparison to be made eg gradients, equations or points. C1 for gradients 1.5 **and** 2 **and** Yes **with** explanation that the gradients are different or states the lines intersect at (0,4) or explanation that interprets common constant term (4) from equations

Q9.

Working	Answer	Mark	Notes
$3x - 2y = 7$ $7x + 2y = 13$ $10x = 20$ $x = 2$ $3\times2 - 2y = 7$ $-2y = 1$ $y = -0.5$	2, −0.5	3	M1 for a correct process to eliminate either x or y (allow one arithmetic error) M1 (dep) for correct substitution of their found variable or an otherwise correct method to eliminate the other variable A1 for 2 and −0.5 oe

Harder simultaneous equations answers

1. x = -3, y = -14
   x = -2, y = -12

2. x = -2, y = +2

3. x = -3, y = -14
   x = -2, y = -12

4. x = 1, y = -2

5.

Working	Answer	Mark	Notes
$y(5y + 24) = 0$ $\frac{-24 \pm \sqrt{(24^2)}}{10}$	$x = 6, y = 0$ $x = -3.6, y = -4.8$	5	M1 for substitution for elimination eg $(2y + 6)^2 + y^2 = 36$ M1 (dep on M1) for expansion eg $4y^2 + 12y + 12y + 36$ (3 out of 4 terms correct) A1 for $4y^2 + 24y + 36 + y^2 = 36$ oe M1 for a correct attempt to solve a 2 or 3 term quadratic equation eg by factorising or correct substitution into a quadratic formula A1 for $x = 6$, $y = 0$ and $x = -3.6$ oe, $y = -4.8$ oe SC: B1 (if M0 scored) for all 4 values mis-associated or one correct pair of values.

6.

Working	Answer	Mark	Notes
$x^2 + (2x + 5)^2 = 25$ $x^2 + 4x^2 + 20x + 25 = 25$ $5x^2 + 20x = 0$ $5x(x + 4) = 0$ $x = 0, x = -4$ $y = 2 \times 0 + 5$ $y = 2 \times -4 + 5$	$x = 0$, $y = 5$ or $x = -4$, $y = -3$	6	M1 $x^2 + (2x + 5)^2$ (= 25) A1 $x^2 + 4x^2 + 10x + 10x + 25$ (= 25) M1 Use of factorisation or correct substitution into quadratic formula or completing the square to solve an equation of the form $ax2 + bx + c = 0$, $a \neq 0$ A1 $x = 0$, $x = -4$ M1 substitution of an x value into an original equation A1 $y = 5$, $y = -3$ correctly matched to x values SC (If M0M0M0 then B1 for one pair (x, y) of correct answers)

7.

Answer	Mark	Notes
$x = 2.87$, $y = -0.87$ and $x = -0.87$, $y = 2.87$	6	M1 for $x^2 + (2 - x)^2 = 9$ M1 for $4 - 4x + x^2$ A1 for $2x^2 - 4x - 5 = 0$ oe 3 term simplified quadratic M1 for a correct method to solve their quadratic Eg $x = \frac{4 \pm \sqrt{(16 - 4 \times 2 \times -5)}}{4}$ A1 for $x = 2.87$, $y = -0.87$ or better A1 for $x = -0.87$, $y = 2.87$ or better Award marks for equivalent algebraic expressions. Apply the same scheme as above for y first.