# Mathematics <br> Paper 1 (Non-Calculator) 

## Higher Tier

Churchill Paper 1C - Marking Guide

Method marks (M) are awarded for a correct method which could lead to a correct answer
Accuracy marks (A) are awarded for a correct answer, having used a correct method, although this can be implied
(B) marks are awarded independent of method

## Churchill <br> Maths

Written by Shaun Armstrong
Only to be copied for use in a single school or college having purchased a licence
$1 \quad 91=7 \times 13$

|  | 71 | 79 | 91 |  |  | B1 | Total 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 32 | 38 | 46 | 50 | 56 | 59 |  |

Median $=\frac{1}{2}(46+50)=48$
If e.g. 46 becomes 60 new median $=\frac{1}{2}(50+56)=53$, up 5
If e.g. 50 becomes 30 new median $=\frac{1}{2}(38+46)=42$, down 6
Largest change $=6$
4
5
6
6.5
B1 Total 1

3 Gradient of $L=\frac{2-0}{(-3)-0}=-\frac{2}{3}$
Gradient of options is $m$ in $y=m x+c$
Gradients $=\begin{array}{llll}-\frac{2}{3} & \frac{2}{3} & -\frac{3}{2} & \frac{3}{2}\end{array}$
Parallel so same gradient, hence $y=4-\frac{2}{3} x$
$y=4-\frac{2}{3} x$
$y=\frac{2}{3} x-\frac{1}{3}$
$y=2-\frac{3}{2} x$
$y=\frac{3}{2} x+1$
B1 Total 1

4 (a) Common difference $=6$ so $n$th term $=6 n+c$

$$
0^{\text {th }} \text { term }=12-6=6
$$

$n$th term $=6 n+6$
$18 n-6$
$12 n+6$
$6 n-6$
$6 n+6$
B1
(b) $\quad u_{3}=8=\frac{16}{u_{2}-2}$

$$
\begin{array}{ll}
8\left(u_{2}-2\right)=16 & \text { M1 } \\
8 u_{2}-16=16 & \\
8 u_{2}=32 & \text { M1 } \\
u_{2}=4 & \\
u_{2}=4=\frac{16}{u_{1}-2} & \\
4\left(u_{1}-2\right)=16 & \\
4 u_{1}-8=16 & \\
4 u_{1}=24 & \text { So, } u_{1}=6 \\
u_{1}=6 & \text { A1 }
\end{array} \text { Total 4 }
$$

$5 \quad 10 \%$ of $50=5$
$40 \%$ of $50=4 \times 5=20$ go into 2 nd round
$25 \%$ of $20=20 \div 4=5$
$75 \%$ of $20=3 \times 5=15$ go into 3 rd round
We know 9 go into 4th round
Fraction of wins in 3rd round $=\frac{9}{15}=\frac{3}{5}$ M1
Percentage wins in 3rd round $=\frac{3}{5} \times 100 \%$

$$
=3 \times 20 \%=60 \%
$$

A1 Total 3

6 (a) $=\frac{1}{2} \times \frac{7}{4}$

$$
=\frac{7}{8}
$$

(b) $=\frac{12}{5} \times \frac{15}{4}$
$=\frac{3}{5} \times \frac{15}{1}$
$=\frac{3}{1} \times \frac{3}{1}$
$=9$
M1 A1 Total 5

7 e.g.



$$
\begin{aligned}
& 110=2 \times 5 \times 11 \\
& 264=2^{3} \times 3 \times 11 \\
& H C F=2 \times 11=22
\end{aligned}
$$

8 Let the number of fourspots used be $x$
She will have used $3 x$ twospots and $2 x$ eightspots
The numbers she has left will be:

| twospot: | $300-3 x$ |  |
| :--- | :--- | :--- |
| fourspot: | $300-x$ | M1 |
| eightspot: | $300-2 x$ |  |
|  |  |  |
|  |  |  |

A1 Total 3

9 (a)

(b) $\frac{2}{15}$
(c) $\frac{2}{5}$

10 Opposite angles add up to $180^{\circ}$ so:

$$
\begin{aligned}
& 4 x+7+4 x-3=180 \\
& 8 x=176 \\
& x=22
\end{aligned}
$$

$4 x-3=4 \times 22-3=88-3=85^{\circ}$
$4 x+7$ is obviously bigger
$3 x+20=3 \times 22+20=66+20=86^{\circ}$
As less than $90^{\circ}$, the opposite angle will be more than $90^{\circ}$
Smallest angle is $85^{\circ}$

| $80^{\circ}$ | $81^{\circ}$ | $83^{\circ}$ | $85^{\circ}$ | B1 | Total 1 |
| :--- | :--- | :--- | :--- | :--- | :--- |

11 (a)


Perimeter $=2+4+2+5+2+3+4=22 \mathrm{~cm}$
A1
(b) $=11^{2}+(7 \times 11)+4=121+77+4=202 \mathrm{~cm}$
(c) $n^{2}+7 n+4=82$
$n^{2}+7 n-78=0$
$(n+13)(n-6)=0$
$n=-13$ or 6
Stage number must be positive so stage 6
Total 6

12 Let no. of LH people $=x$
At first, no. of RH people $=\frac{15}{2} x$
12 RH join so new no. of RH people $=\frac{15}{2} x+12 \quad \mathrm{M} 1$
Ratio is now $9: 1$ so $\quad \frac{15}{2} x+12=9 x \quad$ M1

$$
\begin{aligned}
& 12=\frac{3}{2} x \\
& x=8
\end{aligned}
$$

There are 8 left-handed people

13 (a)

| Width $(w$ cm $)$ | No. fossils | Width $(w c m)$ | Cum. Freq. |
| :---: | :---: | :---: | :---: |
| $0 \leq w<5$ | 0 | $0 \leq w<5$ | 0 |
| $5 \leq w<10$ | 14 | $0 \leq w<10$ | 14 |
| $10 \leq w<15$ | 18 | $0 \leq w<15$ | 32 |
| $15 \leq w<20$ | 13 | $0 \leq w<20$ | 45 |
| $20 \leq w<25$ | 6 | $0 \leq w<25$ | 51 |
| $25 \leq w<30$ | 5 | $0 \leq w<30$ | 56 |
| $30 \leq w<35$ | 3 | $0 \leq w<35$ | 59 |
| $35 \leq w<40$ | 1 | $0 \leq w<40$ | 60 |


(b) From graph, number of fossils with width $\leq 12 \mathrm{~cm}=21$

$$
\begin{aligned}
\text { Using table, no. of fossils } 5 \text { to } 12 & =21 \\
12 \text { to } 25 & =51-21=30 \\
25 \text { to } 40 & =60-51=9
\end{aligned}
$$

$$
\text { Total price } \approx 21 \times £ 6+30 \times £ 20+9 \times £ 50
$$

$$
=126+600+450=£ 1176
$$

[^0]14 First model: $\frac{1}{4}$ off so $\quad \frac{3}{4}$ of original price $=£ 180 \quad$ M1

$$
\begin{aligned}
& \frac{1}{4} \text { of original price }=£ 180 \div 3=£ 60 \\
& \text { original price }=4 \times £ 60=£ 240
\end{aligned}
$$

Second model: $40 \%$ off so $60 \%$ of original price $=£ 180$
$10 \%$ of original price $=£ 180 \div 6=£ 30$
original price $=10 \times £ 30=£ 300$
Difference in pre-sale prices $=300-240=£ 60$
Total 4

15 (a) $\frac{1}{3} \times\left(2.4 \times 10^{17}\right)=0.8 \times 10^{17}=8 \times 10^{16}$
$0.8 \times 10^{17} \quad 8 \times 10^{18} \quad 8 \times 10^{5.67}$
$8 \times 10^{16}$
B1
(b) Underestimate

B1
e.g. 0.32 has been rounded down to $\frac{3}{10}$ which is 0.3
2.387 has been rounded up to 2.39

The percentage that 0.32 has been rounded down is much bigger than the percentage 2.387 has been rounded up
(c) e.g. Jaime's method gives the more accurate estimate The error in rounding 0.32 down to 0.3 is bigger than in rounding it up to $\frac{1}{3}$ (or $0.333 \ldots$ )
Although Jaime's method rounds the 2.387 the other way, for both of them, the error from rounding the 2.387 is much smaller than the error from rounding the 0.32

16
$8^{-\frac{4}{3}}=(\sqrt[3]{8})^{-4}=2^{-4}=\frac{1}{2^{4}}=\frac{1}{16}$


B1 Total 1

17
(a)

$\frac{\sqrt{3}}{3}$
$\frac{\sqrt{2}}{2}$

B1
(b) $\frac{4-\tan 45^{\circ}}{2-\tan 60^{\circ}}=\frac{4-1}{2-\sqrt{3}}$

$$
\begin{aligned}
& =\frac{3}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} \\
& =\frac{3(2+\sqrt{3})}{2^{2}-(\sqrt{3})^{2}} \\
& =\frac{3(2+\sqrt{3})}{4-3} \\
& =\frac{3(2+\sqrt{3})}{1}=3(2+\sqrt{3})=6+3 \sqrt{3}
\end{aligned}
$$

18 (a) $y=\tan x$
(b) $y=\frac{1}{x}$
(c) $y=1+\sin x$
(d) $y=x^{3}-2$

19 (a) By taking $B C$ as the "base" of triangle $A B C$ and $B E$ as the "base" of triangle $A B E$, the two triangles have the same perpendicular height. As the length of $B C$ is one third of the length of $B E$, the base of triangle $A B C$ is one third of the base of triangle $A B E$.
The area of a triangle is $\frac{1}{2} \times$ base $\times$ perpendicular height so the area of triangle $A B C$ is one third of the area of triangle $A B E$.
(b) e.g. Triangle $A B E$ is equilateral so we know

1) angle $A B C=$ angle $A E D\left(=60^{\circ}\right) \quad \mathrm{M} 1$
2) length $A B=$ length $A E$ ( = side length of triangle $A B E$ )

Also, we are told that $B C=(C D=) D E \quad$ M1
We have two pairs of equal sides and the angle between them is also equal, hence congruent by SAS

A1 Total 5

20 Let the number of red counters be $x$
$P\left(1^{\text {st }}\right.$ counter is red $)=\frac{x}{10}$
$P\left(2^{\text {nd }}\right.$ is red given $1^{\text {st }}$ is red $)=\frac{x-1}{9}$
$P($ both red $)=\frac{x}{10} \times \frac{x-1}{9}$
So

$$
\begin{aligned}
& \frac{x}{10} \times \frac{x-1}{9}=\frac{2}{15} \\
& 15 x(x-1)=180 \\
& x(x-1)=12 \\
& x^{2}-x-12=0 \\
& (x+3)(x-4)=0 \\
& x=-3 \text { or } 4
\end{aligned}
$$

$x$ is the number of red counters so can't be negative, hence $x=4$ There are 4 red counters
(a) Area $=\frac{1}{2} \times 2 x \times 3 x \times \sin 60^{\circ}$

$$
=x \times 3 x \times \frac{\sqrt{3}}{2}=\frac{3}{2} x^{2} \sqrt{3}
$$

So

$$
\begin{aligned}
& \frac{3}{2} x^{2} \sqrt{3}=24 \sqrt{3} \\
& \frac{3}{2} x^{2}=24 \\
& x^{2}=\frac{2 \times 24}{3}=16 \\
& x= \pm \sqrt{16}= \pm 4
\end{aligned}
$$

$x$ is a length so it must be positive

$$
x=4
$$

A1
(b) $B C^{2}=A B^{2}+A C^{2}-2 \times A B \times A C \times \cos 60^{\circ}$

$$
x=4 \text { so } A B=8 \text { and } A C=12
$$

$$
B C^{2}=8^{2}+12^{2}-2 \times 8 \times 12 \times \frac{1}{2} \quad \text { M1 }
$$

$$
B C^{2}=64+144-96=112
$$

$$
B C=\sqrt{112}=\sqrt{4 \times 28} \quad M 1
$$

$$
=\sqrt{16 \times 7}=\sqrt{16} \times \sqrt{7}=4 \sqrt{7} \mathrm{~cm} \quad \text { A1 }
$$

22 e.g. $\quad \overrightarrow{X M}=\overrightarrow{X O}+\overrightarrow{O M}=-\overrightarrow{O X}+\overrightarrow{O M}$

$$
\begin{aligned}
& =-6 p+7 p+2 q \\
& =p+2 q
\end{aligned}
$$

A1

As $M$ is the midpoint of $X Y, \overline{M Y}=\overline{X M}=\mathbf{p}+2 \mathbf{q}$

$$
\begin{aligned}
\overrightarrow{N Y}=\overrightarrow{N O}+\overrightarrow{O M}+\overrightarrow{M Y} & =-\overrightarrow{O N}+\overrightarrow{O M}+\overrightarrow{M \bar{Y}} \\
& =-(5 \mathbf{p}+4 \mathbf{q})+7 \mathbf{p}+2 \mathbf{q}+\mathbf{p}+2 \mathbf{q} \\
& =-5 \mathbf{p}-4 \mathbf{q}+7 \mathbf{p}+2 \mathbf{q}+\mathbf{p}+2 \mathbf{q} \\
& =3 \mathbf{p}
\end{aligned}
$$

As $N$ is the midpoint of $Y Z, \overrightarrow{Z Y}=2 \overrightarrow{N Y}=6 \mathbf{p} \quad$ M1
Hence $\overrightarrow{Z Y}=\overrightarrow{O X}$ so $Z Y$ and $O X$ have the same length and are parallel meaning $O X Y Z$ is a parallelogram


[^0]:    Total 6

