# Mathematics <br> Paper 1 (Non-Calculator) 

## Higher Tier

Churchill Paper 1B - Marking Guide

Method marks (M) are awarded for a correct method which could lead to a correct answer
Accuracy marks (A) are awarded for a correct answer, having used a correct method, although this can be implied
(B) marks are awarded independent of method

## Churchill <br> Maths

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$110 \%$ of $82=£ 8.20$
$30 \%$ of $82=3 \times 8.2=£ 24.60$
Sale price $=82-24.60=£ 57.40$
$£ 24.60$
$£ 54.33$
$£ 57.40$
$£ 65.40$
B1
Total 1
$2 \quad 2+3=5$ No
$1+4=5 ; 4+5=9 ; 5+9=14$ No
$2+7=9 ; 7+9=16 ; 9+16=25$ Yes
$1+2=3$ No
$2,3,6,18,108$
$1,4,5,9,10$
$2,7,9,16,25$
$1,2,4,8,16$
B1 Total 1
$3 \quad 3 \times 4=12$ so $0.3 \times 0.4=0.12$
$0.03 \times 0.04=0.0012$

$$
30 \times 0.0004=3 \times 0.004=0.012
$$

$$
0.03 \times 4=0.12
$$

$\begin{array}{ll}0.3 \times 0.4 & 0.03 \times 0.04 \\ 30 \times 0.0004 & 0.03 \times 4\end{array}$
B1
Total 1
$4=\frac{6 a b}{3 a}+\frac{3 a^{3}}{3 a}=2 b+a^{2}$
$6 a b+a^{2} \quad 2 b+a^{2}$
$2 b+3 a^{2} \quad 3 b+a^{2}$
B1 Total 1
$5 \quad 7+4=11$ portions
$55 \div 11=5$ hours per portion
$7 \times 5=35$ hours of badminton
M1
$4 \times 5=20$ hours of basketball
Income $=35 \times 15+20 \times 18$
M1
$35 \times 15=350+350 \div 2$

$$
=350+175
$$

$$
=525
$$

Income $=525+360$

$$
=£ 885
$$

A1 Total 3

6 (a)


60 litres of yellow is mixed with 24 litres of blue
Makes $60+24=84$ litres of green dye B
(b) e.g. Gradient $\approx \frac{100-0}{40-0}=\frac{100}{40}=2.5 \quad$ M1

Hence $Y=2.5 B$
A1
[OR any equivalent form, needn't be explicit Gradient and therefore formula can be slightly different]
(c) $Y: B=2.5: 1$
= $5: 2$
B1 Total 4

7 (a) e.g. 0.215 lies between 0.21 and 0.22
$0.215=\frac{215}{1000}=\frac{43}{200}$
A1
[There are many other correct answers.]
(b) $\frac{1}{4}+\frac{5}{6}+\frac{3}{8}=\frac{6+20+9}{24}=\frac{35}{24} \quad$ M1
mean $=\frac{35}{24} \div 3 \quad$ M1
$=\frac{35}{24} \times \frac{1}{3}=\frac{35}{72} \quad$ A1
Total 5

8


9 (a) The 24 cm equates to 6 lots of the radius

Hence, $r=24 \div 6=4 \mathrm{~cm}$
The perimeter consists of 4 quarter circles $=1$ circle and


4 semicircles $=2$ circles
So perimeter $=3 \times$ circumference of one circle of radius 4 cm
Circumference of 1 circle $=2 \times \pi \times 4=8 \pi$
Perimeter $=3 \times 8 \pi=24 \pi$ A1
(b) Total area consists of 3 circles and the middle "cross"

Area of 1 circle $=\pi \times 4^{2}=16 \pi$ M1
Area of 3 circles $=3 \times 16 \pi=48 \pi$
Consider "cross" as square +4 rectangles
Area of square $=8^{2}=64$
Area of 1 rectangle $=8 \times 4=32$
Area of cross $=64+4 \times 32=192 \quad$ M1
Area of design $=(192+48 \pi) \mathrm{cm}^{2} \quad$ A1
Total 6
$10 x$ will be the height of cloches so $x \geq 12$
Width of cloches will be $60-2 x$
So

$$
\begin{aligned}
& 60-2 x \geq 22 \\
& 60 \geq 22+2 x \\
& 38 \geq 2 x \\
& x \leq 19
\end{aligned}
$$

M1

Hence, $12 \leq x \leq 19$
Total 3
$11=\sqrt{9 \times 3}+\sqrt{16 \times 3}=3 \sqrt{3}+4 \sqrt{3}=7 \sqrt{3}$
$7 \sqrt{3}$
$5 \sqrt{3}$
$4 \sqrt{3}$
$\sqrt{3}$
B1
Total 1
(a) Too high
e.g. $11^{2}=121$ and $12^{2}=144$ so $11.1^{2}$ will be closer to 121 than 144 whereas 135 is closer to 144
(b) Too low
e.g. $\quad 980=9.8 \times 10 \times 10$ which is more than $9.8 \times 9.8 \times 9.8$ or $9.8^{3}$. Hence $\sqrt[3]{980}$ must be more than 9.8
(c) Too high
e.g. $2^{4}=16$ and $3^{4}=81$ so $2.5^{4}$ is between 16 and 81 .

Halfway between 16 and 81 is 48.5 but the value of $x^{4}$ will increase more rapidly as $x$ increases from 2 to 3 so $2.5^{4}$ is likely to be quite a bit below 48.5

M1 A1 Total 4

13 Distance $=$ area under graph

$$
\begin{aligned}
\text { Area } & =\frac{1}{2} \times 8 \times 9+(T-8) \times 9+\frac{1}{2} \times(70-T) \times 9 \\
& =36+9 T-72+315-4.5 T \\
& =4.5 T+279
\end{aligned}
$$

So, $\quad 4.5 T+279=513$

$$
4.5 T=234
$$

$$
9 T=468
$$

$$
T=52
$$

A1
Total 3
$14 \quad \frac{1}{4}=0.25$
$\frac{6}{25}=\frac{24}{100}=0.24$
$\frac{11}{40}=\frac{55}{200}=\frac{275}{1000}=0.275$
$\frac{13}{50}=\frac{26}{100}=0.26$
$0.2 \dot{5}=0.255555 \ldots$ so nearest is 0.26

| $\frac{1}{4}$ | $\frac{6}{25}$ | $\frac{11}{40}$ |
| :--- | :--- | :--- |

## B1 Total 1

15 (a) e.g. Reflection in the line $x=3$
(b) e.g. Reflection in the line through $(-1,6)$ and $(2,0)$

Gradient $=\frac{0-6}{2-(-1)}=-2$
Equation is $\quad y=-2 x+c$
$0=(-2 \times 2)+c$
$c=4$
Transformation is reflection in the line $y=-2 x+4$

16 (a) For size ratio of 3:7, there must be a multiple of 10 balls
For colour ratio of $5: 7$, there must be a multiple of 12 balls
The smallest number of balls will be the LCM of 10 and 12

Multiples of $12=12,24,36,48,60, \ldots$
Smallest multiple of 12 that is a multiple of 10 is 60 M1

Jaime is correct, there must be at least 60 balls in the bucket
A1
(b) e.g. 62 is more than 60 so there must be at least 120 balls in total $120 \div 10=12,3 \times 12=36$ and $7 \times 12=84$ With 120 balls there will be 36 small and 84 large
$120 \div 12=10,5 \times 10=50$ and $7 \times 10=70$
With 120 balls there will be 50 white and 70 black

| Hence: |  | white | black |  |
| :--- | :--- | :---: | :---: | :---: |
|  | small |  |  | 36 |
|  | large |  | 62 | 84 |
|  | 50 | 70 | 120 |  |

Giving:

|  | white | black |  |
| :--- | :---: | :---: | :---: |
| small | 28 | 8 | 36 |
| large | 22 | 62 | 84 |
|  | 50 | 70 | 120 |

The number of small white could be 28
[Other possible answer is 11 when total is 180]

17 (a) $c=\frac{2}{3} a$
(b)


$$
\mathbf{d}=-\frac{1}{3} \mathbf{a}+\mathbf{b}
$$

(c)

$e=\frac{5}{3} \mathbf{a}-\mathbf{b}$
$18 \mathrm{f}(3)=(3 \times 3)+k=9+k$
$\mathrm{ff}(3)=\mathrm{f}(9+k)=3(9+k)+k$
$\begin{array}{cc}=27+3 k+k=27+4 k \\ 27 & +4 k=7\end{array} \quad \mathrm{M} 1$
So, $\quad 27+4 k=7$

$$
4 k=-20
$$

$$
k=-5
$$

A1 Total 3

19 Centre of circle is midpoint of $A B$ which is ( 0,3 )
B1
Radius of circle is half of length $A B=14 \div 2=7$
Distance from $(0,3)$ to $(p, 0)$ must $=7$
Using Pythagoras': $\quad 3^{2}+p^{2}=7^{2}$

$$
\begin{aligned}
& 9+p^{2}=49 \\
& p^{2}=40 \\
& p=\sqrt{40}=\sqrt{4 \times 10}=2 \sqrt{10}
\end{aligned}
$$

20
(a) $\mathrm{P}($ milk $)=\frac{x+(3 x+4)}{\text { total }}=\frac{4 x+4}{\text { total }}$

P (nuts) $=\frac{x+(x-2)}{\text { total }}=\frac{2 x-2}{\text { total }}$
We have $\quad P($ milk $)=3 \times P$ (nuts)

$$
\begin{array}{ll}
\frac{4 x+4}{\text { total }}=3 \times \frac{2 x-2}{\text { total }} & \text { M1 } \\
4 x+4=3(2 x-2) & \text { A1 } \\
4 x+4=6 x-6 & \\
10=2 x & \text { M1 } \\
x=5 &
\end{array}
$$

So,


Total number of chocolates $=19+5+3+13=40$A1
(b) $\qquad$ $\frac{3}{5}$
$\frac{3}{40}$
B1 Total 5

21 (a)

| No. of drinks ( $N$ ) | Frequency | Class <br> width | Frequency <br> density |
| :---: | :---: | :---: | :---: |
| $0 \leq N<10$ | 6 | 10 | 0.6 |
| $10 \leq N<15$ | 4 | 5 | 0.8 |
| $15 \leq N<20$ | 10 | 5 | 2 |
| $20 \leq N<30$ | 7 | 10 | 0.7 |
| $30 \leq N<50$ | 3 | 20 | 0.15 |

e.g. The vertical axis is not consistent - the first two large squares are worth 0.5 , the next two 1.0
The first bar has a height of 0.55 instead of 0.6
(b) Fraction $=\frac{6}{30}=\frac{1}{5}$

Number $=\frac{1}{5} \times 65$ million $=13$ million
(c) e.g. His assumption is not reasonable as his sample is biased because he asked people in a coffee shop who are more likely to drink more coffee.
The value in (b) is too small as in the population there will probably be more people who don't drink many hot drinks.

22
(a) $36^{-\frac{1}{2}}=\frac{1}{\sqrt{36}}=\frac{1}{6}$
$-6 \quad \frac{1}{18}$
$\frac{1}{6} \frac{1}{\sqrt{6}}$

B1
(b) $2^{23} \times 3^{21} \times 6^{-19}=2^{23} \times 3^{21} \times 2^{-19} \times 3^{-19}$

$$
\begin{aligned}
& =2^{23} \times 2^{-19} \times 3^{21} \times 3^{-19} \\
& =2^{4} \times 3^{2} \\
& =16 \times 9 \\
& =144
\end{aligned}
$$

23

$$
\begin{aligned}
\frac{3}{x-3}-\frac{4}{x+1} & =\frac{3(x+1)-4(x-3)}{(x-3)(x+1)} \\
& =\frac{3 x+3-4 x+12}{(x-3)(x+1)} \\
& =\frac{15-x}{(x-3)(x+1)}
\end{aligned}
$$

A1 Total 2

24 e.g.
Let centre be $O$
Let angle $A B C=x$
Let angle $A D C=y$
Angle subtended at centre is twice angle subtended on circumference so:
Angle $A O C=2 y$
Reflex angle $A O C=2 x$
Angles round a point total $360^{\circ}$
so:

$$
2 x+2 y=360^{\circ}
$$



Divide by 2 :
Hence angle $A B C+$ angle $A D C=180^{\circ}$
A1 Total 3
$25 x=\frac{8 \pm \sqrt{(-8)^{2}-4 \times 1 \times(-3)}}{2 \times 1}$
$x=\frac{8 \pm \sqrt{64+12}}{2}$
$x=\frac{8 \pm \sqrt{76}}{2} \quad \sqrt{76}=\sqrt{4 \times 19}=2 \sqrt{19}$
$x=\frac{8 \pm 2 \sqrt{19}}{2}$
$x=4 \pm \sqrt{19}$

