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| **Topic/Skill**  | **Definition/Tips** | **Example****Topic: Further Quadratics**  |
| 1. Quadratic | A quadratic expression is of the form$$ax^{2}+bx+c$$where $a, b$ and $c$ are numbers, $a\ne 0$ | Examples of quadratic expressions:$$x^{2}$$$$8x^{2}-3x+7$$Examples of non-quadratic expressions:$$2x^{3}-5x^{2}$$$$9x-1$$ |
| 2. Factorising Quadratics | When a quadratic expression is in the form $x^{2}+bx+c$ find the two numbers that **add to give b** and **multiply to give c**. | $$x^{2}+7x+10=(x+5)(x+2)$$(because 5 and 2 add to give 7 and multiply to give 10)$$x^{2}+2x-8=(x+4)(x-2)$$(because +4 and -2 add to give +2 and multiply to give -8) |
| 3. Difference of Two Squares | An expression of the form $a^{2}-b^{2}$ can be factorised to give $(a+b)(a-b)$ | $$x^{2}-25=(x+5)(x-5)$$$$16x^{2}-81=(4x+9)(4x-9)$$ |
| 4. Solving Quadratics $(ax^{2}=b)$ | Isolate the $x^{2}$ term and square root both sides.Remember there will be a **positive and a negative solution**. | $$2x^{2}=98$$$$x^{2}=49$$$$x=\pm 7$$ |
| 5. Solving Quadratics $(ax^{2}+bx=0)$ | **Factorise** and then **solve = 0**. | $$x^{2}-3x=0$$$$x\left(x-3\right)=0$$$$x=0 or x=3$$ |
| 6. Solving Quadratics by Factorising $\left(a=1\right)$  | **Factorise** the quadratic in the usual way.**Solve = 0** Make sure the equation = 0 before factorising. | Solve $x^{2}+3x-10=0$Factorise: $\left(x+5\right)\left(x-2\right)=0$$$x=-5 or x=2$$ |
| 7. Quadratic Graph | A ‘**U-shaped**’ curve called a **parabola**.The equation is of the form$y=ax^{2}+bx+c$, where $a$, $b$ and $c$ are numbers, $a\ne 0$. If $a<0$**,** the parabola is **upside down**. | Image result for quadratic graph definition math |
| 8. Roots of a Quadratic  | A root is a **solution**.The roots of a quadratic are the $x$**-intercepts of the quadratic graph**. | Image result |
| 9. Turning Point of a Quadratic | A turning point is the **point where a quadratic turns**.On a **positive parabola**, the turning point is called a **minimum**.On a **negative parabola**, the turning point is called a **maximum**. | Minimum turning pointMaximum turning point |
| 10. Factorising Quadratics when $a\ne 1$ | When a quadratic is in the form$$ax^{2}+bx+c$$1. Multiply a by c = ac2. Find two numbers that add to give b and multiply to give ac.3. Re-write the quadratic, replacing $bx$ with the two numbers you found.4. Factorise in pairs – you should get the same bracket twice5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets. | Factorise $6x^{2}+5x-4$1. $6×-4=-24$2. Two numbers that add to give +5 and multiply to give -24 are +8 and -33. $6x^{2}+8x-3x-4$4. Factorise in pairs: $$2x\left(3x+4\right)-1(3x+4)$$5. Answer = $(3x+4)(2x-1)$ |
| 11. Solving Quadratics by Factorising $\left(a\ne 1\right)$  | **Factorise** the quadratic in the usual way.**Solve = 0** Make sure the equation = 0 before factorising. | Solve $2x^{2}+7x-4=0$Factorise: $\left(2x-1\right)\left(x+4\right)=0$$$x=\frac{1}{2} or x=-4$$ |
| 12. Completing the Square (when $a=1)$ | A quadratic in the form $x^{2}+bx+c$ can be written in the form $(x+p)^{2}+q$1. Write a set of brackets with $x$ in and **half** the value of $b.$2. Square the bracket.3. Subtract $\left(\frac{b}{2}\right)^{2}$and add $c.$4. Simplify the expression.You can **use the completing the square form** to help **find the maximum or minimum** of quadratic graph. | Complete the square of $$y=x^{2}-6x+2$$Answer:$$(x-3)^{2}-3^{2}+2$$$$=(x-3)^{2}-7$$The minimum value of this expression occurs when $(x-3)^{2}=0$, which occurs when $x=3$When $x=3$, $y=0-7=-7$Minimum point = $(3,-7)$ |
| 13. Completing the Square (when $a\ne 1)$ | A quadratic in the form $ax^{2}+bx+c$ can be written in the form **p**$(x+q)^{2}+r$Use the same method as above, but factorise out $a$ at the start. | Complete the square of $$4x^{2}+8x-3$$Answer:$$4\left[x^{2}+2x\right]-3$$$$=4\left[\left(x+1\right)^{2}-1^{2}\right]-3$$$$=4(x+1)^{2}-4-3$$$$=4(x+1)^{2}-7$$ |
| 14. Solving Quadratics by Completing the Square | **Complete the square** in the usual way and **use inverse operations to solve**. | Solve $x^{2}+8x+1=0$Answer:$$\left(x+4\right)^{2}-4^{2}+1=0$$$$\left(x+4\right)^{2}-15=0$$$$\left(x+4\right)^{2}=15$$$$\left(x+4\right)=\pm \sqrt{15}$$$$x=-4\pm \sqrt{15}$$ |
| 15. Solving Quadratics using the Quadratic Formula | A quadratic in the form $ax^{2}+bx+c=0$ can be solved using the formula:$$x=\frac{-b\pm \sqrt{b^{2}-4ac}}{2a}$$Use the formula if the quadratic does not factorise easily. | Solve $3x^{2}+x-5=0$Answer:$a=3, b=1, c=-5$ $$x=\frac{-1\pm \sqrt{1^{2}-4×3×-5}}{2×3}$$$$x=\frac{-1\pm \sqrt{61}}{6}$$$$x=1.14 or-1.47 (2 d.p.)$$ |

**Knowledge Organiser**